5.3 The Lorentz transformation

Chapter belongs to the "Theory of Space"
written by
Dariusz Stanisław Sobolewski.
Http: www.htsengines.com
http: www.theoryofspace.info
E-mail: info@htsengines.com

The Lorentz transformation in a four-dimensional space-time between systems Θ and $\overline{\Theta}$, assuming that system $\overline{\Theta}$ moves with respect to Θ at a velocity $\vec{v} = v\vec{e}_1$, is as follows:

$$\begin{aligned} \bar{t} &= \gamma t - \gamma\beta c^{-1} x_1 \\ \bar{x}_1 &= \gamma x_1 - \gamma\beta ct \\ \bar{x}_2 &= x_2 \\ \bar{x}_3 &= x_3 \end{aligned} \quad \textbf{(1)}$$

, where $\beta = \frac{v}{c} < 1$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}} > 1$

The formula for $\bar{x}_1$ leads directly to the contraction of length which is now interpreted verbatim, thereby its cognitive value is limited. To cope with the problem, we will introduce a postulate which is justified by the space structure:

The postulate of change of orientation of mirror spaces:
Contraction of length at the Lorentz transformation results from the fact that the angle between the normals to boundary hypersurfaces (mirror space) of systems moving with respect to each other is different from zero.

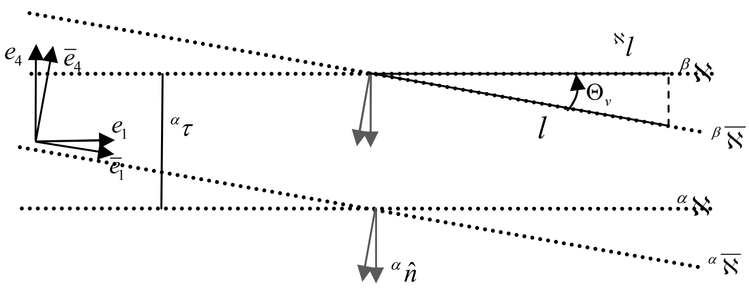

FIG. 1

Directly from the postulate, it follows that the length of a rod, as measured in the system with respect to which it is at rest, is constant.

Therefore, in the figure ${}^{\aleph}l$ denotes a projection of the rod which is at rest in system $\overline{\Theta}$ onto the mirror space of system Θ. This means that measurement of the length of the rod which is in system $\overline{\Theta}$ is a measurement of its projection onto the mirror subspace of system Θ.

From the figure, it follows:

$$
\begin{aligned}
&\cos\Theta_v = \frac{{}^{\aleph}l}{l} = \gamma^{-1} \\
&\left(1-\beta^2\right) + \sin^2\Theta_v = 1 \\
&\sin\Theta_v = \beta
\end{aligned}
\tag{2}
$$

Therefore, using the time dilatation formula $\overline{\Delta t} = \gamma^{-1}\Delta t$, we have:

$$
\begin{aligned}
&\frac{{}^{\aleph}l}{l} = \gamma^{-1} \\
&\frac{\Delta t}{\overline{\Delta t}} = \gamma \\
&\frac{{}^{\aleph}l}{l} * \frac{\Delta t}{\overline{\Delta t}} = 1 = \frac{\dfrac{{}^{\aleph}l}{\overline{\Delta t}}}{\dfrac{l}{\Delta t}} = \frac{c}{c} = 1
\end{aligned}
\tag{3}
$$

Summing up, an observer in system Θ who analyzes a phenomenon consisting in the propagation of a beam of light in system $\overline{\Theta}$ along a rod of the length l takes into consideration[1] time $\overline{\Delta t}$ and a projection of the rod ${}^{\aleph}l$ onto the mirror spaces of system Θ or the length of the rod l and time Δt in system $\overline{\Theta}$. These quantities have been transformed because the angle Θ_v between the normals to the boundary hypersurfaces of systems moving with respect to each other is different from zero.

It is then clearly and obviously seen, that the measurement of the rod's projection on the hypersurfaces of the system leads to introduction of erroneous notions of length contractions and time dilatation.

It must be emphasized expressly that physical coordinate systems are connected with physical objects which define boundary hypersurfaces in their neighborhood. Therefore, special attention should be paid to domains of determinancy of a given local coordinate system which is connected with the given physical object.

Consequently, the following definition of a physical coordinate system is introduced:

Definition of a physical coordinate system

[1] Since the distance to be traversed by the light is „shorter", we have to „lengthen" the time lapse to maintain constant light's speed.

A physical coordinate system, connected with a given physical object which occupies region U, is the map (U, γ) defined on U [2].

In the case referred to above, system $\overline{\Theta}$ was related to a rod and the rod was at rest.

From the postulate of change of orientation for mirror spaces of a system moving at a velocity v in respect of another system, it follows that physical objects which are connected with physical systems have an effect on the geometry of space. The objects referred to above are built of a huge number of space channels which change their orientation thus leading to a change in the orientation of boundary hypersurfaces in the entire system.

To determine the orientation of a space channel moving with respect to system Θ, we will assume that a space channel connecting different mirror subspaces, which is in a region without gravitation[3] is at rest if it is orthogonal to the mirror subspaces of system Θ.

Let us explain that system Θ is a physical coordinate system, therefore, its related space channels $\breve{U}_{\Theta,i}$ define boundary hypersurfaces which are orthogonal to them. Under the circumstances, if we consider space channel $\breve{U}_1$ which is parallel to channels $\breve{U}_{\Theta,i}$, the former is also orthogonal to the boundary hypersurfaces determined by channels $\breve{U}_{\Theta,i}$.

Directly from the figure, it follows that the space channels connected with system $\overline{\Theta}$ form a zero angle with the normals to hypersurfaces ${}^{\alpha}\overline{\aleph}$ and ${}^{\beta}\overline{\aleph}$ and they form angle Θ_v with respect to hypersurfaces ${}^{\alpha}\aleph$ and ${}^{\beta}\aleph$.

A different situation takes place when we consider the motion of a single space channel in system Θ. In that case, depending on velocity, the space channel is inclined at an angle of Θ_v to the normals of boundary hypersurfaces although, this time, their orientation remains unaffected. Under such conditions, the space channel will be carrying out a resultant motion (precession and nutation) which, in quantum mechanics, is equated with waves of matter because of the indeterminancy of its position and momentum.

The issue will be considered in more detail in another chapter, although one can see just now why it is not possible to exactly establish the position of an elementary particle in motion. It will do for the purpose to realize the fact that the position of a moving space channel is determined by means of

[2] See "The effects of gravitation" and "The geometry of the effects of gravitation".

[3] In the chapters to follow we will see that a space channel at rest in respect of an astronomical object which is a source of gravitation field is not orthogonal to boundary hypersurfaces.

three spatial coordinates although a much higher number of such coordinates should be used[4].

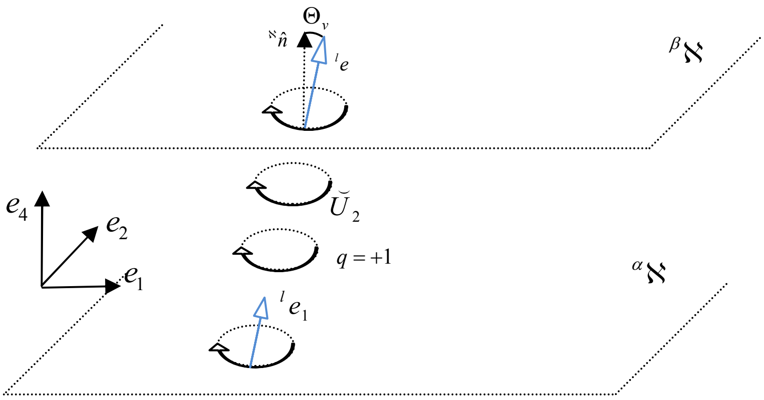

FIG. 2

Shown in the Figure is a space channel, moving at a velocity $\vec{v}$ relative to the physical system Θ :

$$\vec{v} = (\vec{v} \mid e_1) e_1 \quad \textbf{(4)}$$

, for which angle Θ_v is defined as a directed angle $\Theta_v = \angle(^{l}e, e_4)$.

Transformation D_t of coordinates from system $\overline{\Theta}$ to Θ is a transformation resulting from rotation B_{Θ_v} of system $\overline{\Theta}$ in plane $L(e_1, e_4)$ around the invariant plane $L(e_2, e_3)$ and translation C_t in system Θ :

$$D_t = C_t B_{\Theta_v} \quad \textbf{(5)}$$

, where

$$C_t \vec{x} = \vec{x} + \vec{v}t \quad \textbf{(6)}$$

, and

$$B_{\Theta_v} = \begin{bmatrix} \cos\Theta_v & 0 & 0 & \sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \quad \textbf{(7)}$$

, where:

---

[4] Although the issue will be tackled in a different chapter, we can say just now that six space coordinates are required for determining the locations of the ends of space channels alone.

$$\boxed{\begin{aligned}&\sin\Theta_v = \beta\\&\cos\Theta_v = \gamma^{-1} = \sqrt{1-\beta^2}\end{aligned}} \tag{8}$$

In the matrix form, the transformation of the coordinate system takes the following form:

$$D_t = C_t B_{\Theta_v} = \begin{bmatrix} vt \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\Theta_v & 0 & 0 & \sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix} \tag{9}$$

Generally, let us consider velocity in an arbitrary three-dimensional direction:

$$\begin{aligned}&\vec{v} = v_1e_1 + v_2e_2 + v_3e_3 = (\vec{v}\,|\,e_1)e_1 + (\vec{v}\,|\,e_2)e_2 + (\vec{v}\,|\,e_3)e_3 =\\&= \|\vec{v}\|\cos\alpha_1e_1 + \|\vec{v}\|\cos\alpha_2e_2 + \|\vec{v}\|\cos\alpha_3e_3 = \|\vec{v}\|\left(\frac{v_1}{\|\vec{v}\|}e_1 + \frac{v_2}{\|\vec{v}\|}e_2 + \frac{v_3}{\|\vec{v}\|}e_3\right) =\\&= \|\vec{v}\|\hat{v}\end{aligned} \tag{10}$$

and turn the four-dimensional coordinate system in plane $L(\hat{v}, e_4)$ through an angle of Θ_v.

In order to do so[5], let us reduce the turn in plane $L(\vec{v}, e_4)$ to a turn in plane $L(e_1, e_4)$, using two consecutive turns $B_\beta B_\alpha$:

$$B_{\hat{v}} = B_\alpha{}^{-1} B_\beta{}^{-1} B_{\Theta_v} B_\beta B_\alpha \tag{11}$$

Transformation of the coordinate system will take the form:

$$D_t = C_t B_{\hat{v}} = C_t B_\alpha{}^{-1} B_\beta{}^{-1} B_{\Theta_v} B_\beta B_\alpha \tag{12}$$

Turn B_α is a turn of the coordinate system around the invariant hypersurface $L(e_1, e_4)$ through an angle of $\sin\alpha = \dfrac{v_2}{\sqrt{v_2{}^2 + v_3{}^2}}$:

[5] Quaternion algebra may be used as well.

$$B_{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & -\frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(13)}$$

, and then a turn B_{β} around the invariant hypersurface $L(e_2, e_4)$ through an angle of $\sin\beta = \frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|}$:

$$B_{\beta} = \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & \frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(14)}$$

Ultimately, we have:

$$B_{\hat{v}} = {B_{\alpha}}^{-1} {B_{\beta}}^{-1} B_{\Theta_v} B_{\beta} B_{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{-v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & \frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & -\frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\Theta_v & 0 & 0 & \sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & \frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & -\frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(15)}$$

, where:

$$B_{\beta} B_{\alpha} = \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & \frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & -\frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{v_1}{\|v\|} & -\frac{v_2}{\|v\|} & \frac{v_3}{\|v\|} & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ -\frac{\sqrt{{v_2}^2 + {v_3}^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{{v_2}^2 + {v_3}^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{{v_2}^2 + {v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(16)}$$

$$B_{\Theta_v} B_\beta B_\alpha = \begin{bmatrix} \cos\Theta_v & 0 & 0 & \sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & -\frac{v_2}{\|v\|} & \frac{v_3}{\|v\|} & 0 \\ 0 & \frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ -\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{v_1}{\|v\|}\cos\Theta_v & -\frac{v_2}{\|v\|}\cos\Theta_v & \frac{v_3}{\|v\|}\cos\Theta_v & \sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ -\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \tag{17}$$

$$B_\beta^{-1} B_{\Theta_v} B_\beta B_\alpha = \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & -\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|}\cos\Theta_v & -\frac{v_2}{\|v\|}\cos\Theta_v & \frac{v_3}{\|v\|}\cos\Theta_v & \sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ -\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{{v_1}^2}{\|v\|^2}\cos\Theta_v + \frac{{v_2}^2+{v_3}^2}{\|v\|^2} & -\frac{v_1 v_2}{\|v\|^2}\cos\Theta_v + \frac{v_1 v_2}{\|v\|^2} & \frac{v_1 v_3}{\|v\|^2}\cos\Theta_v - \frac{v_1 v_3}{\|v\|^2} & \frac{v_1}{\|v\|}\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ v_1\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}\cos\Theta_v - v_1\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2} & -v_2\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}\cos\Theta_v - \frac{{v_1}^2}{\|v\|^2}\frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & v_3\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}\cos\Theta_v + \frac{{v_1}^2}{\|v\|^2}\frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|}\sin\Theta_v \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} =$$

$$= \begin{bmatrix} 1 + \frac{{v_1}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1}{\|v\|}\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & 0 \\ v_1\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}(\cos\Theta_v - 1) & -v_2\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}\cos\Theta_v - \frac{{v_1}^2}{\|v\|^2}\frac{v_2}{\sqrt{{v_2}^2+{v_3}^2}} & v_3\frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|^2}\cos\Theta_v + \frac{{v_1}^2}{\|v\|^2}\frac{v_3}{\sqrt{{v_2}^2+{v_3}^2}} & \frac{\sqrt{{v_2}^2+{v_3}^2}}{\|v\|}\sin\Theta_v \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \tag{18}$$

$$B_{\hat{v}}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{-v_2}{\sqrt{v_2^2+v_3^2}} & 0\\ 0 & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1}{\|v\|}\sin\Theta_v\\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & 0\\ v_1\frac{\sqrt{v_2^2+v_3^2}}{\|v\|^2}(\cos\Theta_v-1) & -v_2\frac{\sqrt{v_2^2+v_3^2}}{\|v\|^2}\cos\Theta_v-\frac{v_1^2}{\|v\|^2}\frac{v_2}{\sqrt{v_2^2+v_3^2}} & v_3\frac{\sqrt{v_2^2+v_3^2}}{\|v\|^2}\cos\Theta_v+\frac{v_1^2}{\|v\|^2}\frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{\sqrt{v_2^2+v_3^2}}{\|v\|}\sin\Theta_v\\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v\end{bmatrix}=$$

$$=\begin{bmatrix}1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1}{\|v\|}\sin\Theta_v\\ -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_3^2}{v_2^2+v_3^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_2^2}{\|v\|^2(v_2^2+v_3^2)} & \frac{v_2v_3}{v_2^2+v_3^2}-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v-\frac{v_1^2v_2v_3}{\|v\|^2(v_2^2+v_3^2)} & -\frac{v_2}{\|v\|}\sin\Theta_v\\ \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_2v_3}{v_2^2+v_3^2}-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v-\frac{v_1^2v_2v_3}{\|v\|^2(v_2^2+v_3^2)} & \frac{u_2^2}{v_2^2+v_3^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)} & \frac{v_3}{\|v\|}\sin\Theta_v\\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v\end{bmatrix}= \qquad \textbf{(19)}$$

$$=\begin{bmatrix}1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1}{\|v\|}\sin\Theta_v\\ -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & 1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_2v_3}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_2}{\|v\|}\sin\Theta_v\\ \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_2v_3}{\|v\|^2}(\cos\Theta_v-1) & 1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_3}{\|v\|}\sin\Theta_v\\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v\end{bmatrix}$$

The following reductions were used in Formula (19):

$$\frac{v_2v_3}{v_2^2+v_3^2}-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v-\frac{v_1^2v_2v_3}{\|v\|^2(v_2^2+v_3^2)}=\frac{v_2v_3}{v_2^2+v_3^2}\left(1-\frac{v_1^2}{\|v\|^2}\right)-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v=$$
$$=\frac{v_2v_3}{v_2^2+v_3^2}\left(\frac{\|v\|^2}{\|v\|^2}-\frac{v_1^2}{\|v\|^2}\right)-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v=\frac{v_2v_3}{v_2^2+v_3^2}\left(\frac{v_2^2+v_3^2}{\|v\|^2}\right)-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v= \qquad \textbf{(20)}$$
$$=\frac{v_2v_3}{\|v\|^2}-\frac{v_2v_3}{\|v\|^2}\cos\Theta_v=\frac{v_2v_3}{\|v\|^2}(1-\cos\Theta_v)$$

$$\frac{v_3^2}{v_2^2+v_3^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_2^2}{\|v\|^2(v_2^2+v_3^2)}=\frac{(v_1^2+v_2^2+v_3^2)v_3^2}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_2^2}{\|v\|^2(v_2^2+v_3^2)}=$$
$$=\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_3^2}{\|v\|^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_2^2}{\|v\|^2(v_2^2+v_3^2)}=\frac{v_1^2(v_2^2+v_3^2)}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_3^2}{\|v\|^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v= \qquad \textbf{(21)}$$
$$=\frac{v_1^2}{\|v\|^2}+\frac{v_3^2}{\|v\|^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v=1-\frac{v_2^2}{\|v\|^2}+\frac{v_2^2}{\|v\|^2}\cos\Theta_v=1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v-1)$$

$$\frac{v_2^2}{v_2^2+v_3^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)}=\frac{\|v\|^2v_2^2}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)}=$$
$$=\frac{(v_1^2+v_2^2+v_3^2)v_2^2}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)}=\frac{v_1^2v_2^2}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_2^2}{\|v\|^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v+\frac{v_1^2v_3^2}{\|v\|^2(v_2^2+v_3^2)}= \qquad \textbf{(22)}$$
$$=\frac{v_1^2(v_2^2+v_3^2)}{\|v\|^2(v_2^2+v_3^2)}+\frac{v_2^2}{\|v\|^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v=\frac{v_1^2}{\|v\|^2}+\frac{v_2^2}{\|v\|^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v=1-\frac{v_3^2}{\|v\|^2}+\frac{v_3^2}{\|v\|^2}\cos\Theta_v=$$
$$=1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v-1)$$

Transformation $B_{\hat{v}}$ takes the final form:

$$B_{\hat{v}} = \begin{bmatrix} 1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1}{\|v\|}\sin\Theta_v \\ -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2}{\|v\|}\sin\Theta_v \\ \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_3}{\|v\|}\sin\Theta_v \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \quad \textbf{(23)}$$

Matrix $B_{\hat{v}}$ determinant is equal to unity because:

$$\begin{aligned} |B_{\hat{v}}| &= |B_\alpha^{-1} B_\beta^{-1} B_{\Theta_v} B_\beta B_\alpha| = |B_\alpha^{-1}||B_\beta^{-1}||B_{\Theta_v}||B_\beta||B_\alpha| = \\ &= |B_\alpha|^{-1}|B_\beta|^{-1}|B_{\Theta_v}||B_\beta||B_\alpha| = |B_{\Theta_v}| = 1 \end{aligned} \quad \textbf{(24)}$$

The reversed transformation $B_{\hat{v}}^{-1}$ equals the following[6]:

$$B_{\hat{v}}^{-1} = \left(B_\alpha^{-1} B_\beta^{-1} B_{\Theta_v} B_\beta B_\alpha\right)^{-1} = B_\alpha^{-1} B_\beta^{-1} B_{\Theta_v}^{-1} B_\beta B_\alpha = B_\alpha^T B_\beta^T B_{\Theta_v}^T B_\beta B_\alpha$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{-v_2}{\sqrt{v_2^2+v_3^2}} & 0 \\ 0 & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & \frac{u_3}{\sqrt{v_2^2+v_3^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & -\frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(25)}$$

$$\begin{bmatrix} \cos\Theta_v & 0 & 0 & -\sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & \frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & 0 \\ 0 & -\frac{v_2}{\sqrt{v_2^2+v_3^2}} & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Consecutively, we have the following:

$$B_{\Theta_v}^T B_\beta B_\alpha = \begin{bmatrix} \cos\Theta_v & 0 & 0 & -\sin\Theta_v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\Theta_v & 0 & 0 & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|} & -\frac{v_2}{\|v\|} & \frac{v_3}{\|v\|} & 0 \\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & 0 \\ -\frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{v_2^2+v_3^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{v_2^2+v_3^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{v_1}{\|v\|}\cos\Theta_v & -\frac{v_2}{\|v\|}\cos\Theta_v & \frac{v_3}{\|v\|}\cos\Theta_v & -\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{v_2^2+v_3^2}} & \frac{v_2}{\sqrt{v_2^2+v_3^2}} & 0 \\ -\frac{\sqrt{v_2^2+v_3^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{v_2^2+v_3^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{v_2^2+v_3^2}} & 0 \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \quad \textbf{(26)}$$

[6] Matrix $B_{\hat{v}}$ is not orthogonal, therefore, it is necessary to repeat the calculations.

$$B_{\beta}{}^{T} B_{\Theta_v}{}^{T} B_{\beta} B_{\alpha} = \begin{bmatrix} \frac{v_1}{\|v\|} & 0 & -\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|} & 0 & \frac{v_1}{\|v\|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_1}{\|v\|}\cos\Theta_v & -\frac{v_2}{\|v\|}\cos\Theta_v & \frac{v_3}{\|v\|}\cos\Theta_v & -\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ -\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|} & -\frac{v_1}{\|v\|}\frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{v_1}{\|v\|}\frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{v_1{}^2}{\|v\|^2}\cos\Theta_v + \frac{v_2{}^2+v_3{}^2}{\|v\|^2} & -\frac{v_1 v_2}{\|v\|^2}\cos\Theta_v + \frac{v_1 v_2}{\|v\|^2} & \frac{v_1 v_3}{\|v\|^2}\cos\Theta_v - \frac{v_1 v_3}{\|v\|^2} & -\frac{v_1}{\|v\|}\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ v_1\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v - v_1\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2} & -v_2\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v - \frac{v_1{}^2}{\|v\|^2}\frac{v_2}{\sqrt{u_2{}^2+u_3{}^2}} & v_3\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v + \frac{v_1{}^2}{\|v\|^2}\frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & -\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|}\sin\Theta_v \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} = \quad \textbf{(27)}$$

$$= \begin{bmatrix} 1+\frac{v_1{}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1}{\|v\|}\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ v_1\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}(\cos\Theta_v - 1) & -v_2\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v - \frac{v_1{}^2}{\|v\|^2}\frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & v_3\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v + \frac{v_1{}^2}{\|v\|^2}\frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & -\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|}\sin\Theta_v \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix}$$

$$B_{\hat{v}}{}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{-v_2}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ 0 & \frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{u_3}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+\frac{v_1{}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1}{\|v\|}\sin\Theta_v \\ 0 & \frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & \frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & 0 \\ v_1\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}(\cos\Theta_v - 1) & -v_2\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v - \frac{v_1{}^2}{\|v\|^2}\frac{v_2}{\sqrt{v_2{}^2+v_3{}^2}} & v_3\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|^2}\cos\Theta_v + \frac{v_1{}^2}{\|v\|^2}\frac{v_3}{\sqrt{v_2{}^2+v_3{}^2}} & -\frac{\sqrt{v_2{}^2+v_3{}^2}}{\|v\|}\sin\Theta_v \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \quad \textbf{(28)}$$

Ultimately:

$$B_{\hat{v}}{}^{-1} = \begin{bmatrix} 1+\frac{v_1{}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1}{\|v\|}\sin\Theta_v \\ -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_2{}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_2}{\|v\|}\sin\Theta_v \\ \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_3{}^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_3}{\|v\|}\sin\Theta_v \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \quad \textbf{(29)}$$

Transformation D_t of the coordinates of system $\overline{\Theta}$ to Θ will be in the form:

$$D_t = C_t B_{\hat{v}} \quad \textbf{(30)}$$

, which gives the following after taking into consideration Formula (23):

$$D_t\bar{x} = C_t B_{\bar{v}}\bar{x} =$$

$$= \begin{bmatrix} v_1 t \\ v_2 t \\ v_3 t \\ 0 \end{bmatrix} + \begin{bmatrix} 1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1}{\|v\|}\sin\Theta_v \\ -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2}{\|v\|}\sin\Theta_v \\ \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_3}{\|v\|}\sin\Theta_v \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix} \qquad \textbf{(31)}$$

Reversed transformation D_t^{-1}:

$$D_t^{-1} = B_{\bar{v}}^{-1} C_t^{-1} =$$

$$= \begin{bmatrix} 1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1}{\|v\|}\sin\Theta_v \\ -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_2}{\|v\|}\sin\Theta_v \\ \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_3}{\|v\|}\sin\Theta_v \\ \frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v \end{bmatrix} \left(\begin{bmatrix} -(\bar{v} \mid e_1)t \\ -(\bar{v} \mid e_2)t \\ -(\bar{v} \mid e_3)t \\ 0 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) \qquad \textbf{(32)}$$

It follows from the formulas above that the transformation of coordinates, in the Theory of Space, does not change the distance between two arbitrary points. Time intervals, at the passage from one system to another, are invariant as well, therefore, transformation of spatial coordinates may be extended in a simple manner by transformation of time:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ t \end{bmatrix} = \begin{bmatrix} (\bar{v} \mid e_1)t \\ (\bar{v} \mid e_2)t \\ (\bar{v} \mid e_3)t \\ 0 \\ 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 1+\frac{v_1^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_1}{\|v\|}\sin\Theta_v & 0 \\ -\frac{v_1 v_2}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_2^2}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2}{\|v\|}\sin\Theta_v & 0 \\ \frac{v_1 v_3}{\|v\|^2}(\cos\Theta_v - 1) & -\frac{v_2 v_3}{\|v\|^2}(\cos\Theta_v - 1) & 1+\frac{v_3^2}{\|v\|^2}(\cos\Theta_v - 1) & \frac{v_3}{\|v\|}\sin\Theta_v & 0 \\ -\frac{v_1}{\|v\|}\sin\Theta_v & \frac{v_2}{\|v\|}\sin\Theta_v & -\frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \\ \bar{t} \end{bmatrix} \qquad \textbf{(33)}$$

, whereas a reversed transformation:

$$
\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \\ \bar{t} \end{bmatrix} =
\begin{bmatrix}
1+\frac{v_1^{\,2}}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_1}{\|v\|}\sin\Theta_v & 0 \\
-\frac{v_1v_2}{\|v\|^2}(\cos\Theta_v-1) & 1+\frac{v_2^{\,2}}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_2v_3}{\|v\|^2}(\cos\Theta_v-1) & \frac{v_2}{\|v\|}\sin\Theta_v & 0 \\
\frac{v_1v_3}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_2v_3}{\|v\|^2}(\cos\Theta_v-1) & 1+\frac{v_3^{\,2}}{\|v\|^2}(\cos\Theta_v-1) & -\frac{v_3}{\|v\|}\sin\Theta_v & 0 \\
\frac{v_1}{\|v\|}\sin\Theta_v & -\frac{v_2}{\|v\|}\sin\Theta_v & \frac{v_3}{\|v\|}\sin\Theta_v & \cos\Theta_v & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ t \end{bmatrix} - \begin{bmatrix} (\vec{v} \mid e_1)t \\ (\vec{v} \mid e_2)t \\ (\vec{v} \mid e_3)t \\ 0 \\ 0 \end{bmatrix} \right) \qquad \textbf{(34)}
$$

Using Formulas (8), in which it should be assumed that $v = \|v\|$, and after substituting these to (33) we obtain:

$$
\begin{aligned}
x_1 &= v_1t + \left(1+\frac{v_1^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)\bar{x}_1 - \frac{v_1v_2}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_2 + \frac{v_1v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_3 + \frac{v_1}{c}\bar{x}_4 \\
x_2 &= v_2t - \frac{v_1v_2}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_1 + \left(1+\frac{v_2^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)\bar{x}_2 - \frac{v_2v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_3 - \frac{v_2}{c}\bar{x}_4 \\
x_3 &= v_3t + \frac{v_1v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_1 - \frac{v_2v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)\bar{x}_2 + \left(1+\frac{v_3^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)\bar{x}_3 + \frac{v_3}{c}\bar{x}_4 \\
x_4 &= -\frac{v_1}{c}\bar{x}_1 + \frac{v_2}{c}\bar{x}_2 - \frac{v_3}{c}\bar{x}_3 + \gamma^{-1}\bar{x}_4 \\
t &= \bar{t}
\end{aligned} \qquad \textbf{(35)}
$$

Reversed transformation takes the form:

$$
\begin{aligned}
\bar{x}_1 &= \left(1+\frac{v_1^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)(x_1-v_1t) - \frac{v_1v_2}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_2-v_2t) + \frac{v_1v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_3-v_3t) - \frac{v_1}{c}x_4 \\
\bar{x}_2 &= -\frac{v_1v_2}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_1-v_1t) + \left(1+\frac{v_2^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)(x_2-v_2t) - \frac{v_2v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_3-v_3t) + \frac{v_2}{c}x_4 \\
\bar{x}_3 &= v_3t + \frac{v_1v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_1-v_1t) - \frac{v_2v_3}{\|v\|^2}\left(\gamma^{-1}-1\right)(x_2-v_2t) + \left(1+\frac{v_3^{\,2}}{\|v\|^2}\left(\gamma^{-1}-1\right)\right)(x_3-v_3t) - \frac{v_3}{c}x_4 \\
\bar{x}_4 &= \frac{v_1}{c}(x_1-v_1t) - \frac{v_2}{c}(x_2-v_2t) + \frac{v_3}{c}(x_3-v_3t) + \gamma^{-1}x_4 \\
\bar{t} &= t
\end{aligned} \qquad \textbf{(36)}
$$

Let us write down special cases of transformation for velocities situated on the plain $L(e_1, e_2)$. For this kind of cases, transformations may be determined by substituting the following:

$$v_3 = 0 \qquad \textbf{(37)}$$

, in Formulas (35) and (36).

$$
\begin{aligned}
x_1 &= v_1 t + \left(1 + \frac{v_1^{\ 2}}{\|v\|^2}\left(\gamma^{-1} - 1\right)\right)\bar{x}_1 - \frac{v_1 v_2}{\|v\|^2}\left(\gamma^{-1} - 1\right)\bar{x}_2 + \frac{v_1}{c}\bar{x}_4 \\
x_2 &= v_2 t - \frac{v_1 v_2}{\|v\|^2}\left(\gamma^{-1} - 1\right)\bar{x}_1 + \left(1 + \frac{v_2^{\ 2}}{\|v\|^2}\left(\gamma^{-1} - 1\right)\right)\bar{x}_2 - \frac{v_2}{c}\bar{x}_4 \\
x_3 &= \bar{x}_3 \\
x_4 &= -\frac{v_1}{c}\bar{x}_1 + \frac{v_2}{c}\bar{x}_2 + \gamma^{-1}\bar{x}_4 \\
t &= \bar{t}
\end{aligned}
\tag{38}
$$

, and:

$$
\begin{aligned}
\bar{x}_1 &= \left(1 + \frac{v_1^{\ 2}}{\|v\|^2}\left(\gamma^{-1} - 1\right)\right)\left(x_1 - v_1 t\right) - \frac{v_1 v_2}{\|v\|^2}\left(\gamma^{-1} - 1\right)\left(x_2 - v_2 t\right) - \frac{v_1}{c}x_4 \\
\bar{x}_2 &= -\frac{v_1 v_2}{\|v\|^2}\left(\gamma^{-1} - 1\right)\left(x_1 - v_1 t\right) + \left(1 + \frac{v_2^{\ 2}}{\|v\|^2}\left(\gamma^{-1} - 1\right)\right)\left(x_2 - v_2 t\right) + \frac{v_2}{c}x_4 \\
\bar{x}_3 &= x_3 \\
\bar{x}_4 &= \frac{v_1}{c}\left(x_1 - v_1 t\right) - \frac{v_2}{c}\left(x_2 - v_2 t\right) + \gamma^{-1}x_4 \\
\bar{t} &= t
\end{aligned}
\tag{39}
$$

In a special case, for:

$$
\begin{aligned}
v_2 &= 0 \\
v_3 &= 0
\end{aligned}
\tag{40}
$$

we have:

$$
\begin{aligned}
x_1 &= v_1 t + \left(1 + \frac{v_1^{\ 2}}{\|v\|^2}\left(\gamma^{-1} - 1\right)\right)\bar{x}_1 + \frac{v_1}{c}\bar{x}_4 = v_1 t + \gamma^{-1}\bar{x}_1 + \frac{v_1}{c}\bar{x}_4 \\
x_2 &= \bar{x}_2 \\
x_3 &= \bar{x}_3 \\
x_4 &= -\frac{v_1}{c}\bar{x}_1 + \gamma^{-1}\bar{x}_4 \\
t &= \bar{t}
\end{aligned}
\tag{41}
$$

, and:

$$
\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \\ \bar{t} \end{bmatrix}
=
\begin{bmatrix}
\cos\Theta_v & 0 & 0 & -\sin\Theta_v & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\sin\Theta_v & 0 & 0 & \cos\Theta_v & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\left(
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ t \end{bmatrix}
-
\begin{bmatrix} \left(\vec{v} \mid e_1\right)t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\right)
\tag{42}
$$

Summing up, it is stated that conventional formulas for transformation from system $\overline{\Theta}$ to Θ result from an erroneous assumption concerning the structure of space, leading to equating objects from system $\overline{\Theta}$ and their projections onto boundary hypersurfaces of system Θ .

The table below specifies the projections of physical quantities of system $\overline{\Theta}$ onto boundary hypersurfaces of system Θ :

Physical quantity	Transformation	Projection
l	$l\gamma^{-1}$	$l\cos\Theta_v$
Δt	$\Delta t\gamma^{-1}$	$\Delta t\cos\Theta_v$